PREDICTING THE PRICES OF ELECTRICITY DERIVATIVES ON THE ENERGY EXCHANGE

Štěpán Kratochvíl, Oldřich Starý*

1. Introduction

This paper has several objectives. The first is a comparison of the results of long-term spot price forecast and analysis to identify the benefit of extending the mean-reversion model to the jump-diffusion model as the jumps are quite frequent and specific by their big size compared to other commodities. Another objective is to compare forecast accuracy with the regime-switching model, which uses another approach to modelling jumps. Forecasting of spot prices uses innovative filtering methods, which refine the prediction by removing abnormal calibration data. The main objective is to compare the evaluated future prices from these spot prices using direct future price modelling.

Before the 1990s, the electricity market was regulated by state authorities. There was no trading in electricity, because electricity prices were set as a function of generation, transmission and distribution costs. In recent years, a deregulation process has taken place in many countries, and this has opened up electricity markets for trading and has interconnected adjacent markets; see Álvaro (2012). The main aim of the deregulation was to introduce competition to generation and supply activities (though not to transmission and distribution, which remain natural monopolies). Electricity prices are now determined by the interaction between supply (generators) and demand (suppliers who trade with energy and ultimately sell energy to customers). For more information, see Chemišinec et al. (2010).

The trading has grown since the deregulation process began. Electricity is no longer used only for power, but increasingly in the form of derivatives for hedging or speculation on price rises or falls; see Jílek (2002). This is connected with the need for high-precision prediction of electricity prices, because prediction accuracy is linked with the profits or losses made by market participants; see Viehmann (2011).

Many works have been written on the theme of predicting electricity prices. Predicting models are used not only for predicting spot electricity prices, but in all areas of finance. However, as we will explain in the next section, electricity behaves differently from other financial commodities.

There are many approaches to electricity price predictions. The basic models are mean-reversion models based on Brownian motion, used by Bierbrauer et al. (2007), Blanco et al (2001), García-Martos, and Rodríguezand Sánchez (2011). This model is further extended to jump diffusion, which refines the results, used by Blanco et al. (2001), Meyer-Brandis (2008), and Uhrig-Homburg and Seifert (2007). The advantages of these two models are combined in the regime-switching model. This model switches between several modes, which are often represented by the mean-reversion and jump-diffusion models, presented by Deng (2000) and Weron (2008).

In our paper, we first introduce these models for predicting prices, and sketch the advantages and drawbacks of the models. Next, we show methods for estimating the parameters of the model and filtering methods for better prediction results. Finally, we apply the models to EEX (the Central European energy market) data.

The paper is organised as follows. Section 2 describes the main characteristics of electric energy as mean-reversion, high volatility, seasonality and frequent jumping behaviour. Section 3 presents the models used for prediction. Section 4 explains methods for estimating the parameters of the models. Section 5 describes our data set, makes an economic analysis of the data and defines the filtering methods. Section 6 presents the results of spot price predictions. Section 7 calculates the future price from the spot price. Section 8 concludes the results.

2. Specific Characteristics of Electricity Prices

Electricity behaves differently from other financial commodities, see Carol (1999) or Huisman et al. (2007). The major reason for this is the non-storability of electricity. Once it has been generated, electricity must be consumed immediately: at the same time, however, there must be enough energy in the grid to meet the customers' needs. This leads to the need for real balancing of supply, which is the reason why price jumps occur. The behaviour of electricity prices must be captured in the prediction model, so it is important to have a very good understanding of the behaviour.

Since the balance between the supply and the demand runs in real time (due to the non-storability), the behaviour of the prices is cyclical, depending on recurrent fluctuations in the demand and the supply. These cycles are short-term or long-term. Short cycles are caused by the level of the population's economic activity (phases of the day: day vs. night mode). Long-term cycles are based on the contrasting seasonal climatic conditions and the length of day and night. This seasonal behaviour has an effect on electricity prices. Especially in the winter months, the prices of electricity are seen to increase due to higher consumption for heating. In this paper, we will not model seasonality separately, but will include this behaviour in the models.

Another feature of electricity that makes it different from other financial derivatives is its high volatility. This feature is caused by the non-storability, a finite transport network, and balancing in real time. These properties lead to large fluctuations in the price of electricity. A further reason for the volatile behaviour is that electricity is a necessary commodity (for heating, lighting, appliances, etc.) and is also highly weather-dependent.

The price of electricity has a tendency to return to a long-term mean value. However, the price of electricity differs from other commodities in its rate of return. While other commodities take weeks to return to the long-term mean, electricity returns much more rapidly, in a matter of hours or days. Again due to the non-storability, the reaction to changes on the market needs to be rapid. Increases in demand are met by electric power generators that generate comparatively expensive electricity (due to higher generation costs). When a normal level of demand returns, the generators are switched off and the price of electricity will also return to a steady level.

The jumps are the most distinctive feature of the electricity price. Price jumps occur very frequently and they are of very short duration due to their height. Generally speaking, the lower the jump (resulting in lower price levels), the longer the jump will be. High jumps normally last for a maximum of 1-3 days. We can see that the jump behaviour is very closely connected with the mean-reversion behaviour.

This behaviour is caused by power outages or overloading of the electrical system. These jumps are for very short periods of time, and unfortunately we are not able to predict them. Again, this behaviour is caused by the non-storability and the need to balance electricity supply and demand in real time.

3. Models and Their Descriptions

Energy derivatives markets differ significantly from other financial markets. The main distinctive features are the higher volatility, the seasonal behaviour of spot prices, the mean-reverting behaviour, and the occurrence of price jumps. The higher volatility and the jump behaviour of electricity are caused by the non-storability. Since electricity cannot be stored, the slightest drop in demand or supply will cause a step change in the price. These characteristics have to be captured in the predicting model.

There are many models for predicting electricity prices. The basic division is into deterministic and stochastic models. We will concentrate on stochastic models here.

Next, we will have to determine how to approach price modelling and price prediction. There are two basic approaches according to Schindlmayr (2004):

Market model for future prices: Instead of modelling the spot price and calculating the future price from the spot price, the future price can be modelled directly. For this approach, the Black-Scholes model, the Black model or the Monte Carlo simulation method can be used. The drawback of this approach is that the future price does not reflect the price behaviour of the underlying asset at an hourly or daily valuation.

Model spot prices: Models of this type focus on modelling spot prices on the basis of historical data (hourly or daily valuation). The future price is then derived from the modelled spot price.

We will focus on the second type of approach, because this type is more suitable for us. Finally, we will compare the results with the first approach. Next, we will define mean-reversion models, jump-diffusion models and regime-switching models, which will be used in the paper.

3.1 Mean-reversion model

This model is widely used by both electricity vendors and electricity producers. It is a basic model, simple to understand, simple for estimating the parameters, and easy to use. The reason for using this model is that electricity prices tend to return quickly to their mean value. The model is based on Brownian motion, but improves that approach with a mean-reversion component. We will now provide a mathematical description of the model.

$$dY_t = \gamma \left(\mu - Y_t \right) dt + \sigma dW_t, \ \ t \geq 0 \tag{1}$$

Where $\left(Y_t \right)_{t \geq 0}$ represents the non-seasonal log-price process, $\left(W_t \right)_{t \geq 0}$ Brownian motion (Wiener process), γ indicates the speed of the transition (return) to the mean value, μ indicates long-term mean and σ indicates volatility.

After analysing the patterns, we can see that they consist of two components. The first one provides the return to the mean value (after a lower or higher value caused by jumps or volatility fluctuation); this is done by taking the difference between the current value and the process mean, multiplied by the speed of its return. Here, we can see the importance of the factor. This factor must be large enough to ensure a sufficiently rapid return to the mean, which is specific for electricity. The second component models oscillation of electricity prices.

This model contains a benefit, but also a major weakness. It fails to describe and thus predict the jumps that occur frequently in the spot prices of electricity. This shortcoming can be removed by extending the mean-reversion model to the jump-diffusion model.

3.2 Jump-diffusion model

The jump-diffusion model is based on the mean-reversion model. However, it offers one improvement. Besides components that return to the mean and model random fluctuation, it has a third component: the jump. It is created by the Poisson distribution multiplied by the height of the jump (2). Thanks to this component, the jump-diffusion model can better model the evolution of the spot prices of electricity. It adapts better to the natural behaviour of the electricity market, where jumps occur quite often and have to be included in the model.

$$dY_t = \gamma \left(\mu - Y_t \right) dt + \sigma dW_t + q dN_t, \ \ t \geq 0 \tag{2}$$

Equation (2) describes the jump-diffusion model, where $\left(Y_t \right)_{t \geq 0}$ represents the non-seasonal log-price process, $\left(W_t \right)_{t \geq 0}$ is Brownian motion (Wiener process) and γ, μ and σ are real constants, $\left(N_t \right)_{t \geq 0}$ is a homogeneous Poisson process. The height of a jump q has a log-normal distribution with the mean υ and the variance τ^2.

By adding the jump component, we refine the modelled value of the spot price, but there is a slow return to the mean value. We can reduce this shortcoming by switching between multiple states. The first state represents the basic mode (a mean-reversion

process) and other modes represent the jump parts of the model. This model will be shown in the next section.

3.3 Regime-switching model

The advantage of the regime-switching model is that it consists of two (or three) separate components (modes), each of which has a different process. The observed jump can be explained by a transition to another mode. The regime-switching model is usually assumed to follow a time-homogeneous hidden Markov chain (see Cipra (2008)) with $k \in N$ possible modes representing k states of the system.

3.4 Regime-switching with two independent states

The regime-switching model with two independent states distinguishes between a basic mode $(R_t = 1)$ and a peaks mode $(R_t = 2)$, where $(R_t)_{t \in T}$ represents a time-homogeneous hidden Markov chain. An observable stochastic process $(Y_t)_{t \in T}$ is now represented in the form $Y_t = Y_{t, R_t}$, $t \in T$, where processes $(Y_{t,1})_{t \in T}$ and $(Y_{t,2})_{t \in T}$ are mutually independent. $Y_{t,i}$ expresses the current regime i at the time t. Transitions between modes can be described by the transition matrix π of the hidden Markov chain, R contains the probability $p_{i,j}$ of switching from the regime i at the time t to the mode j at the time $t+1$:

$$\pi = \left(p_{i,j}\right)_{i,j=1,2} = \left(P\left(R_{t+1} = j | R_t = i\right)\right)_{i,j=1,2} = \begin{pmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{22} & p_{22} \end{pmatrix} \tag{3}$$

Finally, we will specify the processes $Y_{t,1}$ and $Y_{t,2}$. Taking into account the typical behaviour of spot electricity prices, it seems reasonable to use the mean-reversion process as the basic mode $(R_t = 1)$. For the peaks mode $(R_t = 2)$, it is difficult to assign the appropriate process. We will assign it an independent, identically distributed realisation probability distribution F. Log-normal distribution is the best candidate.

As a result, we consider the following two stochastic processes for the normal mode (4) and the peaks mode (5).

$$Y_{t,1} = c + \emptyset Y_{t-1,1} + \epsilon_t, \qquad t \in N \tag{4}$$

$$Y_{t,2} \approx F, \qquad t \in N \tag{5}$$

This would be better for modelling the spot price of electricity in accordance its characteristics.

3.5 Regime-switching model with three independent states

This model is based on the previous model with two independent states. We will now define the three modes that are used: (1) the basic mode $(R_t = 1)$, for modelling the mean-reverting process of electricity prices, (2) the initial jump mode $(R_t = 2)$, for

modelling a sudden increase or decrease in the prices, and (3) the reverse jump mode $(R_t = 3)$, for describing the return of the prices to a normal level after the occurrence of a jump in the reverse jump mode, in order not to remain in the jump mode. The process will be described by the mean-reversion process in the basic mode and the Poisson process in the initial mode and the reverse jump modes, where the direction of the initial jump process is opposite to that of the reverse jump. The description of the processes will be:

$$Y_t = \begin{cases} \emptyset Y_t + c + \epsilon_t, & R_t = 1 \left(normal \right) \\ Y_{t-1} + \xi_t, & R_t = 2 \left(initial\ jump \right) \\ Y_{t-1} - \xi_t, & R_t = 3 \left(reverse\ jump \right) \end{cases} \tag{6}$$

Here, $\epsilon \sim N(0, \sigma^2)$ represents an innovation in the basic mode and $\xi_t \sim N(v, \tau^2)$ represents an innovation in the jump modes, which are the jumps. Of course, log-normal distribution of the jump modes can be replaced with another alternative distribution.

4 Estimation parameters

This section is probably the most important part of the whole work. Proper parameter setting has the greatest impact on the accuracy of the results. If the best, most detailed and most comprehensive model is chosen for the data, but the wrong parameters are set, the results obtained from the model will be on a insufficient level. The second most important factor is choosing the right model (described above). We will therefore discuss three different models that will be compared with each other. The resulting models are the mean-reversion model, the jump-diffusion model and the regime--switching model.

There are many methods for optimal parameter setting. We have chosen a linear regression method and a maximum likelihood method.

4.1 Mean-reversion model

The parameters of this model are estimated using a linear regression method according to Smith (2010). First of all, we will recall the mathematical description of the model.

$$dY_t = \gamma \left(\mu - Y_t \right) dt + \sigma dW_t, \, t \geq 0 \tag{7}$$

This general description is edited to a form suitable for estimating the parameters. This form is then used for the jump-diffusion model.

$$X_t = \alpha + \left(1 - \beta \right) X_{t-1} + \varepsilon_t \tag{8}$$

$$X_t = \alpha + X_{t-1} - \beta X_{t-1} + \varepsilon_t \tag{9}$$

$$X_t - X_{t-1} = \alpha - \beta X_{t-1} + \varepsilon_t \tag{10}$$

The mean value is represented by $\frac{\alpha}{\beta}$ and the mean-reversion rate γ is represented by β. Now, we can proceed to estimate the parameters using a linear regression.

The linear regression is performed on the data $X_t - X_{t-1}$ against X_{t-1}. As a result, we get the equation of the regression line in the form:

$$y = a + bx + \varepsilon_t \tag{11}$$

The parameters of the mean-reversion model can be estimated from this equation.

$$\alpha = \frac{a}{\Delta t} \tag{12}$$

$$\beta = -\frac{b}{\Delta t} \tag{13}$$

$$\sigma = \frac{sd(\varepsilon_t)}{\sqrt{\Delta t}} \tag{14}$$

4.2 Jump-diffusion model

First, let us recall the equation for the jump-diffusion model:

$$dY_t = \alpha(\mu - Y_t)dt + \sigma dW_t + q dN_t, \; t \geq 0 \tag{15}$$

This equation is again edited to a more convenient form.

$$X_t - X_{t-1} = \alpha - \beta X_{t-1} + \varepsilon_t + q dN_t \tag{16}$$

As was mentioned above, this model is based on mean-reversion and extends it for the jump component. However, this model allows jumping in one direction only, whereas there are jumps up and down in our data. We therefore extend the model with a next part: a jump part modelled by the Poisson process:

$$X_t - X_{t-1} = \alpha - \beta X_{t-1} + \varepsilon_t + q_u dN_{u,t} - q_d dN_{d,t}, \; t \geq 0 \tag{17}$$

It can be seen that it is now a basic part and the other two parts are step parts. We will therefore classify the data into three groups. The first group will be within the spot price from zero to the value for the first data quartile (39.1); the second group will be the middle part from the first quartile to the third quartile (69.41); the last group will be from the third quartile up. The first and the third groups will be described using the Poisson process and the central part will be described using the mean-reversion process. Figure 1 shows a separation of the groups. We will now divide the data and

apply the method described in the previous section for estimating the parameters for the mean-reversion model to the data falling into the second group.

For the first and the third groups, we have to estimate the Poisson process and the jump width (in our case, the height of the jump). The Poisson process depends on a single parameter: the intensity of the event (in our case, the intensity of a jump). The equation for the probability density Poisson process can be expressed as follows:

$$f(k, \lambda) = \frac{\lambda^k * e^{-\lambda}}{k!} \tag{18}$$

Figure 1
Representations of quartiles. Separation of the jump parts from the mean part
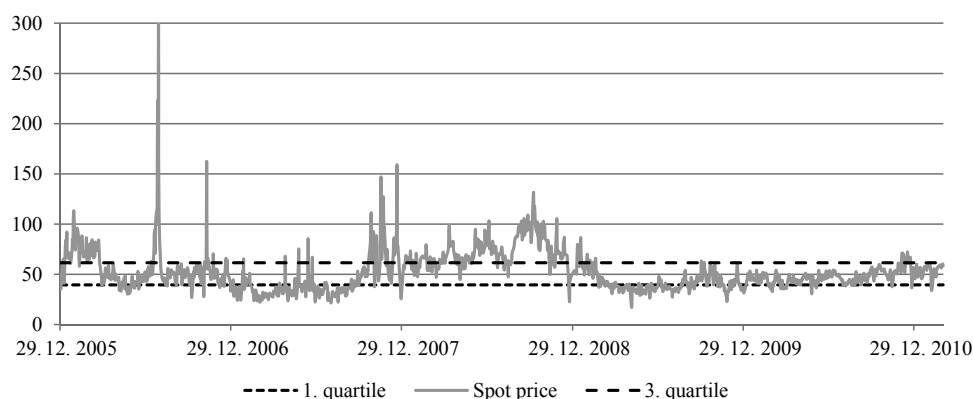
where the intensity parameter is estimated as follows.

$$\lambda = \frac{number\ of\ detected\ jumps}{total\ amount\ of\ data} \tag{19}$$

With this parameter, we can forecast the prices in the first and the third groups in such a way that the result of the jump process is deducted from the lower limit for the first group and added to the upper limit for the third group. Log-normal distribution was chosen for the modelling of the height of the jumps with υ as the average of the prices in the group and τ^2 as the variance of the prices.

This procedure will also be used in the next section, in estimating the parameters of the jump modes of the regime-switching model.

4.3 Regime-switching model

In this model, we will consider three possible modes. The basic mode, which will be modelled using the mean-reversion model, while the initial jump mode and the reversion jump mode will be modelled using the Poisson process with log-normal distribution. We determined the boundaries of the jumps in the same way as for the jump-diffusion model in the previous section (Figure 1). The mean and standard deviation parameters will be calculated from the filtered data (data that remain in the area bounded by the boundary lines between the modes).

Other unknown parameters are the probability transition matrix and the parameters of the mean-reversion model. These are the most important parameters, so we have to determine them with the greatest accuracy. We therefore chose the maximum likelihood method for this estimate, i.e., the EM algorithm, presented by Weron et al. (2010).

As the name implies, the EM algorithm consists of two superposed successive phases. The phase E is used to determine the probability of being at the time t conditional that the process is in the mode i and for the knowledge of data until the time t. The phase M is used to refine the unknown parameters θ. Each iteration of the algorithm consists of the following two phases with a number of iterations that is chosen empirically so that when the unknown parameters do not change significantly after the next iteration, the evaluation will quit. Now we will sketch the algorithm.

First, we will define the vector the of unknown parameters: $\theta^{(0)} = \left(\alpha^{(0)}, \beta^{(0)}, \sigma^{(0)}, P^{(0)} \right)$. The top right index determines the iteration in which the parameters were computed. In addition, we will show the phase E. This phase is divided into two steps. The probabilities are filtered out in the first step and the probabilities are smoothed in the second step.

Phase E:

Filtering:

$$P(R_t = i \mid x_t; \theta^{(n)}) = \frac{P(R_t = i \mid x_{t-1}; \theta^{(n)}) f(x_t \mid R_t = i; x_{t-1}; \theta^{(n)})}{\sum_{i=1}^{3} P(R_t = i \mid x_{t-1}; \theta^{(n)}) f(x_t \mid R_t = i; x_{t-1}; \theta^{(n)})} \tag{20}$$

where $f(x_t \mid R_t \quad i; x_t \quad ; \quad^{(\ )})$ is the probability density of the mode i. Next we will compute:

$$P(R_{t+1} = i \mid x_t; \theta^{(n)}) = \sum_{j=1}^{3} p_{ji}^{(n)} P(R_t = j \mid x_t; \theta^{(n)}) \tag{21}$$

as long as $P(R_T = i \mid x_T; \theta^{(n)})$ is not calculated. Here, T is the size of the calibrating data and $p_{ji}^{(n)}$ notes the probability of transition from the state j to the state i in the *n-th* iteration.

Smoothing:

In this step, we will proceed in reverse from the time T to 0. We will therefore choose $t = T\text{-}1, T\text{-}2, ..., 1$.

$$P\left(R_t = i | x_T ; \theta^{(n)}\right) = \sum_{j=1}^{3} \frac{P(R_t = i \mid x_t ; \theta^{(n)}) P(R_{t+1} = j \mid x_T ; \theta^{(n)}) p_{ji}^{(n)}}{P(R_{t+1} = i \mid x_t ; \theta^{(n)})} \tag{22}$$

The probability density for the normal mode (mean-reversion process) used in the filtering step looks like this:

$$f\left(x_t | R_t = i ; x_{t-1} ; \theta^{(n)}\right) = \frac{1}{\sqrt{2\pi}\sigma^{(n)}} * exp\left\{-\frac{\left(x_t - \left(1 - \beta^{(n)}\right)x_{t-1} - \alpha^{(n)}\right)^2}{2\left(\sigma^{(n)}\right)^2}\right\} \tag{23}$$

with the mean $\alpha_t + \left(1 - \beta^{(n)}\right)x_{t-1}$ and the standard deviation $\sigma^{(n)}$.

By this, we have acquired the vector probabilities of being in the mode i at the time t, with the knowledge of the data until the time t. These values are further used to refine the unknown parameters θ in the phase M.

Phase M:

In this phase, we will refine the vector of the unknown parameters in each iteration. The necessary relationship is listed below:

$$\alpha = \frac{\sum_{t=2}^{T}\left[P\left(R_t = i | x_T ; \theta^{(n)}\right)\left(x_t - (1 - \beta)x_{t-1}\right)\right]}{\sum_{t=2}^{T}\left[P\left(R_t = i | x_T ; \theta^{(n)}\right)\right]} \tag{24}$$

$$\beta = \frac{\sum_{t=2}^{T}\left[P\left(R_t = i | x_T ; \theta^{(n)}\right)x_{t-1}B_1\right]}{\sum_{t=2}^{T}\left[P\left(R_t = i | x_T ; \theta^{(n)}\right)x_{t-1}B_2\right]} \tag{25}$$

$$B_1 = x_t - x_{t-1} - \frac{\sum_{t=2}^{T}\left[P\left(R_t = i | x_T ; \theta^{(n)}\right)\left(x_t - x_{t-1}\right)\right]}{\sum_{t=2}^{T}\left[P\left(R_t = i | x_T ; \theta^{(n)}\right)\right]} \tag{26}$$

$$B_2 = \frac{\sum_{t=2}^{T}\left[P\left(R_t = i | x_T ; \theta^{(n)}\right)x_{t-1}\right]}{\sum_{t=2}^{T}\left[P\left(R_t = i | x_T ; \theta^{(n)}\right)\right]} - x_{t-1} \tag{27}$$

$$\sigma^2 = \frac{\sum_{t=2}^{T}\left[P\left(R_t = i | x_T ; \theta^{(n)}\right)\left(x_t - \alpha - (1 - \beta)x_{t-1}\right)\right]^2}{\sum_{t=2}^{T}\left[P\left(R_t = i | x_T ; \theta^{(n)}\right)\right]} \tag{28}$$

Here, we have refined all the parameters of the normal mean-reversion process. Other relationships are used to determine the matrix of transition probabilities:

$$
p_{ij}^{(n+1)} = \frac{\sum_{t=2}^{T} P(R_t = j, R_{t-1} = i \mid x_T; \theta^{(n)})}{\sum_{t=2}^{T} \left[P\left(R_{t-1} = i \mid x_T; \theta^{(n)}\right) \right]} = \cdot
$$

$$
= \frac{\sum_{t=2}^{T} P(R_t = j \mid x_T; \theta^{(n)}) \dfrac{p_{ij}^{(n)} P(R_{t-1} = i \mid x_{t-1}; \theta^{(n)})}{P(R_t = j \mid x_{t-1}; \theta^{(n)})}}{\sum_{t=2}^{T} \left[P\left(R_{t-1} = i \mid x_T; \theta^{(n)}\right) \right]}
$$

(29)

After the phase M, the complete vector of the unknown parameters θ is therefore estimated. This means that after a sufficient number of iterations, the θ stops change noticeably, the model is established and we can go on with the spot price prediction.

5. Data

We work with daily data from the EEX (see Figure 2). We chose the PHELIX future for the 2012 option with an underlying annual base load, see Deng and Oren (2006). This option is labelled F1BY, and the period is determined by the time of maturity Jan 2012. F1BY Jan 2012 futures first appeared on the stock exchange on 29 December 2005. The data is listed until 1 March 2011. This future was chosen due to suitable and sufficient liquidity, large traded volumes and behaviour similar to that of the PXE. The forecasting of the spot prices used the daily PHELIX spot index. Figure 2 presents in graphic form the data, on parts of which the model will be calibrated and the year 2010 will be predicted by the model. Descriptive statistics are presented in Table 1. The detailed data analysis performed before estimating the model used for forecasting the prices is a key element for achieving accurate results. The model (model type) should be adapted to the data, because it is not necessarily the most complex and sophisticated model that will give the best results. For example, if we predict a jump on the basis of data in which jumps do not occur, we will get worse results.

Figure 2
Measured spot and future prices
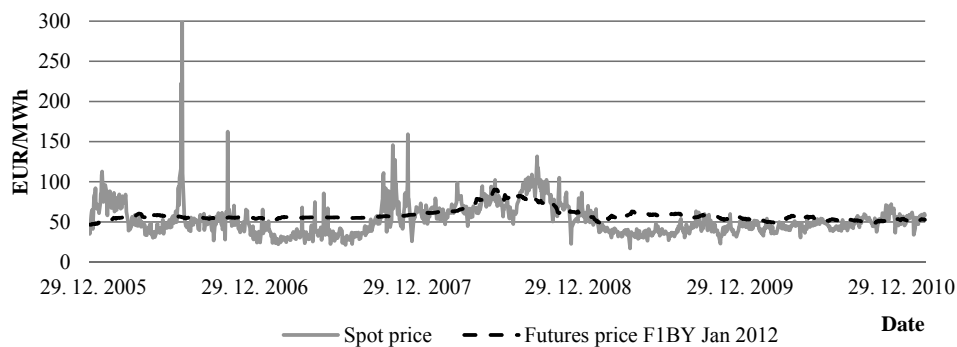
Table 1

Descriptive statistics of the hourly spot and future electricity prices of the EEX

	Mean	Median	Std dev	Skewness	Kurtosis	Min	Max	25% qtl	75% qtl
Spot	53.01	48.78	20.24	2.81	22.14	17.06	301.54	39.43	61.56
Futures	58.19	55.75	7.93	1.95	3.54	46.58	90.50	53.84	58.90

The data show the specific features of electricity prices and the presence of jumps. It can be seen that a mean-reversion model would not be adequate for modelling the spot prices, because this model does not model price jumps. Furthermore, a significant rate of return from jumps to the stable price level can also be seen here. This feature may reduce the accuracy of the jump-diffusion model. A prerequisite for our work is to obtain the best results from the regime-switching model. In the following section, we will therefore find out whether we have fulfilled this requirement.

If we focus on the data, we notice some abnormal periods. The first abnormal period is the steep rise in spot prices up to EUR 300/MWh and the subsequent fall in the value settled in July 2007. This behaviour was caused by several factors. The first factor was the extremely hot weather, which caused the system load to rise by 500 MW. Another factor was a switch to provisional lines at Hradec, which were repaired after a storm in May 2007. Due to corrections and inspections, four other lines were shut down. Another factor was that switch gear in Slovenia was turned off due to a fire, and this led to an increase in consumption from the Czech Republic to Austria. This led to overloads at Hradec and turned off the switchgear. The subsequent domino effect accounts for further outages. Another price increase can be observed on 7 Nov 2006, which was caused by electricity outages in France, Austria, Belgium, Italy and Spain. An outage lasting for one hour was caused by launching of a large ship from the docks into the North Sea. Other abnormal data can be observed between the last quarter of 2007 and the end of 2008. These jumps and the higher average prices of electricity were caused by the global economic crisis. This crisis was caused in July 2007 by the crisis on the US mortgage market. Because of the interconnectedness of markets, the crisis quickly spread throughout the world. The situation calmed down in 2008, and this again led to a price increase in October 2008. The end of the crisis (at least on the energy exchanges) can be observed at the beginning of 2009. Finally, it should be noted that there are initially unusually high data when a derivative is opened. These data may be explained as unstable behaviour due to the beginning of trading followed by gradual stabilisation in March 2006.

In the previous section, we outlined the models that we use to predict the spot price and the method for calibrating the parameters of these models. We also pointed out what data are used for the calibration, price prediction and analysis of the behaviour of prices in time. It has been shown that it will probably not be ideal to use all the data from 2006-2009 for calibrating the models for the prediction of 2010. We will therefore make simulations for three sets of different input data:

1. Use all the data, and try a "blind" prediction.
2. Use only the last year before the predicting year (2010) for the calibration.

3. Make an economic analysis of the data and perform filtering. This means that we will discard from the set of calibrating data any data that are abnormal (i.e., atypical for the product) by their nature (economic background, accompanied by the evolution of prices). This will result in a data set that will be more useful for the prediction purposes (see Figure 3).

Figure 3
Filtered spot calibration data
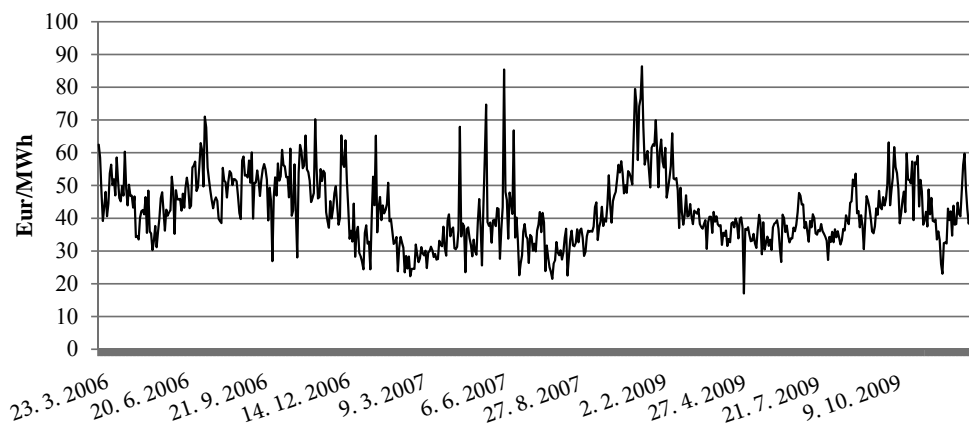
6. Results of predicting spot prices

For the statistical processing results, we will use the indicators of the average deviation (MAE and RMSE) and the envelope method for calculating the 95% confidence interval.

The envelope method involves repeating predictions of new values to get 39 samples. From the samples, we choose the maximum and minimum values for each day. These values form the envelope. If newly predicted data occur between the envelopes, it can be said that we have reached a two-sided 95% confidence interval, presented in Moller and Waagepetersen (2004).

A comparison of the values of different input data models shows that the best results were achieved using the jump-diffusion model (Figure 4). By contrast, the worst average deviation was for the results achieved using the regime-switching model. This is a surprising result, since this model was evaluated as the best in the original analysis and the graph output does not seem so bad (Figure 5). However, it achieved a good result in the test envelope, where the mean-reversion model yields the worst values.

We also see how suitable the original data analysis was. Through this original data analysis, another two data samples were chosen from the original calibration data set, on the basis of which the calibrated model predicts the data with much greater precision (it reduced the average deviation by EUR 3–5/MWh). The results are presented in Table 2.

Table 2:

Results of spot price prediction. The last line shows values exceeding the envelopes.

| Model	Mean-reversion			Jump-diffusion			Regime-switching		
Input data	All data	2009	filtered	All data	2009	filtered	All data	2009	filtered
MAE	12.58	8.03	8.28	9.94	7.23	6.87	12.92	9.19	9.36
RMSE	15.64	10.34	10.61	14.59	9.14	9.08	17.96	11.83	12.73
Envelope method	7	10	6	3	2	2	4	1	2

Figure 4

Spot price predicted by the jump-diffusion model calibrated on filtered data. The black dashed line is the measured data, and the gray line is the predicted data. The bottom axis shows the date and the left axis shows the price in EUR/MWh.
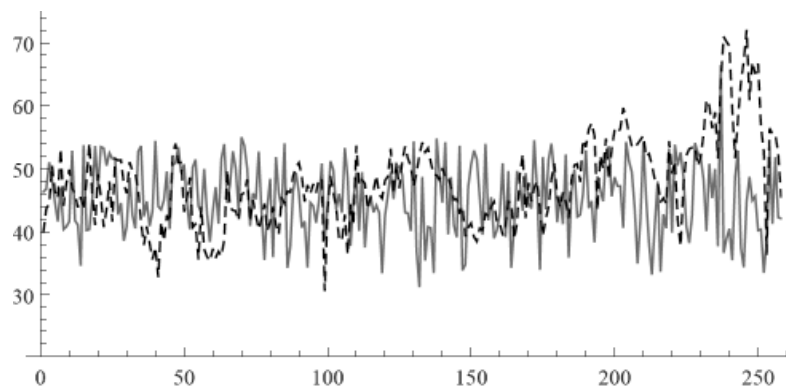
Figure 5

Spot price predicted by the regime-switching model calibrated on filtered data. The black dashed line is the measured data, and the gray line is the predicted data. The bottom axis shows the date and the left axis shows the price in EUR/MWh.
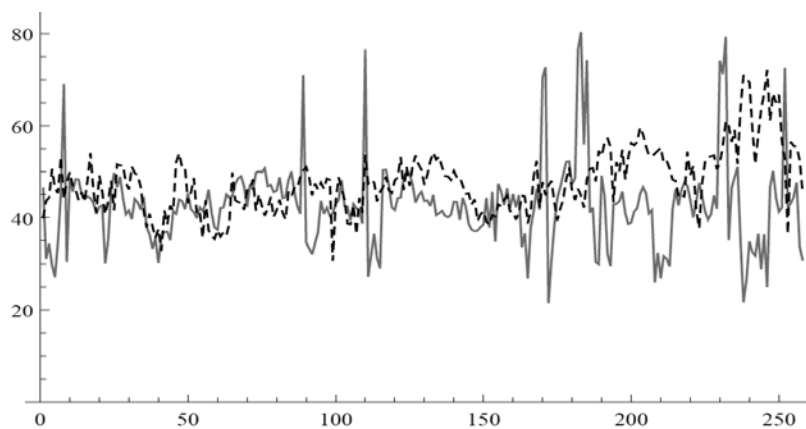
7. Results of prediction of future prices

After successful prediction of the spot price, we will now calculate futures prices. We will use the general formula, presented in Horáček (2008).

$$F(t,T) = S(t) * e^{r_f(T-t)}, \tag{30}$$

where $S(t)$ is the spot price of the underlying asset (electricity), is the annual risk-free interest rate, t indicates the time of the contract, and T is the contract expiry time.

However, the data structure for spot prices differs significantly from the data structure for futures prices. The spot price reflects properties of electricity such as non-storability, balancing in real time and other features described in Section 3, which are reflected in the spot price in the form of frequent and high jumps up and down, and a variable mean. In futures prices, these symptoms are completely missing, and there are only slight fluctuations around the mean value. However, Equation (31) cannot smooth spot prices to ensure future prices. The data therefore need to be smoothed manually. To do this, we choose two procedures – Monte Carlo simulation, and the moving average. (10,000 simulations of the spot price forecasting were made for the Monte Carlo simulation, and the best moving average length of 400 days was analysed for the moving average.)

Table 3 shows that better results were achieved using the moving average. This is what we expected, since Monte Carlo simulation smooths data to the mean, while the moving average method is better for capturing a growing/declining trend. The lowest deviation values were obtained for the data predicted by the mean-reversion model using the moving average method. The highest deviation value was obtained using the Monte Carlo simulation for the data predicted by the jump-diffusion model. This was already explained in the previous section by the influence of the step behaviour of the input data.

Table 3
Results of future price prediction. Average deviations are displayed in the table.

Model	Mean-reversion		Jump-diffusion		Regime-switching	
Measure	MAE	RMSE	MAE	RMSE	MAE	RMSE
Monte Carlo simulation	3.8	5.4	6.47	9.67	4.76	6.13
Moving average	3.15	3.62	2.94	3.48	4.14	4.46

These results can be compared with direct future price forecasting. The mean-reversion model is used for this, because of the behaviour of the future prices, where price jumps are not so frequent and the volatility is much weaker (see Table 1). Using this model, we get the result of MAE, which is 7.38 and RMSE, which is 8.81. It can be seen that better results are achieved by forecasting the spot prices and then modelling the future prices from them.

8. Conclusions

We have described the models used for predicting electricity prices on energy exchanges. These are mean-reversion, jump-diffusion and regime-switching models. For these models, we then searched for procedures for estimating the optimal parameters. We used the following methods: linear regression the for mean-reversion model and the basic part of the jump-diffusion model, the maximum likelihood estimation (EM algorithm) for the basic mode of the regime-switching model, and a Poisson process for jumps of the jump-diffusion and regime-switching model.

Next, we made an analysis of the input data for calibrating the model, and the analysis resulted in three data sets for the calibration. The first set consists of all the data from 2006 to 2009. The second set is data for the year 2009. The last set of data consists of filtered samples of data between 2006 and 2009, eliminating abnormal spot prices.

We then predicted new spot price values, and compared these data with actual measured values. The MAE value ranged from EUR 7.23 to EUR 12.92/MWh, and the average price of EUR 53.01/MWh implies a deviation from 14 to 25%. From both the MAE and the RMSE, the best results were achieved for the jump-diffusion model and the calibration of the 2009 data and the filtered data.

From these data, we calculated the future price by smoothing the spot price using a Monte Carlo simulation and the moving average method. The deviation calculated from the actual future prices was measured in the range from EUR 2.94 to 6.47/MWh, which at the average price of EUR 58.19/MWh implies a deviation of 5-11%. Among the smoothing methods for spot prices, the moving average method achieved the best results, and among the spot price prediction models (one of which was used to calculate future prices) the mean-reversion model achieved the best results. We proved that this approach to modelling the future prices using spot price forecast is more accurate than direct future price forecasting.

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PREDICTING THE PRICES OF ELECTRICITY DERIVATIVES ON THE ENERGY EXCHANGE

Abstract: There is a need to focus on electricity derivative trading, because this is an important and expanding field. The aim of this paper is long-term forecasting of the daily futures prices. Two approaches were used for this, namely the use of spot price forecasting to model the future prices and forecasting future prices directly. We will show on an EEX case study that better results can be achieved by the first approach, where we use mean-reverting, jump-diffusion and regime-switching models for spot price forecasting. The best results of spot price forecasting are achieved by the jump-diffusion model, where we will present the benefit of the use of filtered calibration data.

Keywords: electricity derivatives, energy exchange, predicting prices, estimation of the parameters, data filtering

JEL Classification: C, G1, Q4